## Sequence alignment heuristics

## Lecture 11

## Local Alignment

o More often, we want to find the local regions of high similarity, rather than the overall sequence scores

- The time is quadratic, and the result is highly influenced by the scoring scheme


## Simple Exclusion method. Baeza-Yates, Perelberg (BYP)

- Suppose P matches a substring T1 of T with at most k errors (insertions, deletions, substitutions). Then T1 must contain at least 1 interval of length $r=M /(k+1)$ that exactly matches one of the $r$-length substrings of $P$.
- Proof. If we partition P into consecutive r-length regions, and align P to T 1 , then there would be $\mathrm{k}+1$ sub-alignments. If each of these sub-alignments were to contain at least 1 error, then there would be more than $k$ errors in total.


## Alignment using BYP

- Find a local alignment of P to T with an optimal score, but with an additional constraint that there would not be more than $k$ errors between $P$ and the aligned region T1 of T.

1. Partition P into $\mathrm{k}+1$ consecutive substrings
2. Find the set of the possible locations of alignment $P$ to the part of T , by exactly matching each of the $\mathrm{k}+1$ substrings to T
3. Extend each found match from both ends to the full length of P using dynamic programming (computing $2 \mathrm{k}+1$ - strip around the main diagonal). If the resulting alignment has up to $k$ errors, report it

## BYP expected running time

- Proven that:
- Algorithm BYP runs in $\mathrm{O}(\mathrm{N})$ time for $\mathrm{k}<=\mathrm{O}\left(\mathrm{M} / \log _{\sigma} \mathrm{M}\right)$, where $\sigma$ is the size of the alphabet
- For a DNA sequence $(\sigma=4)$ of length $64, k$ can be as high as $64 / 4=16$ or $25 \%$
- For a protein sequence ( $\sigma=20$ ) of length $400, k$ can be as high as 400/2=200 or 50\%
- In practice, $\mathrm{r}=\mathrm{M} /(\mathrm{k}+1)$ should be at least 9 for DNA and at least 5 for proteins to be efficient. This is because the asymptotic $\mathrm{O}\left(\mathrm{M} / \log _{0} \mathrm{M}\right)$ contains an unknown constant
- For DNA of length 100 - no more than 9 errors, or 9\%


## Optimal global alignment

- Given 2 input strings S1 and S2, and the scoring matrix, find an alignment with the maximum possible score
- The scoring matrix includes also the gap scoring scheme
- The optimality of an alignment heavily depends on the scoring matrix


## Example. Scoring matrix 1



Total best score: 0

## Example. Scoring matrix 2



Total best score: 1

## Choosing the scoring scheme

o C-DNA example

- In Eukaryotes, a protein-coding gene is made of alternating exons (expressed sequences) and introns (intervening sequences), which do not code for a protein
- The number of exons is generally modest (4-20), but the lengths of introns can be huge comparing to the length of introns



## Choosing the scoring scheme

o C-DNA example

- If we extract m-RNA from the working cell at any stage of its working cycle, and then we transcribe it backwards into DNA by using viral reverse transcriptase, we obtain c-DNA
- Now we want to align the obtained c-DNA with the region of genome in order to locate the gene and to study the mechanism of splicing



## Choosing the scoring scheme for c-DNA alignment

- If the spaces are penalized on the unit bases, that would align c-DNA substrings close together rather than allowing large gaps corresponding to introns
- The number of mismatches in the aligned regions should not be large, since these regions are just a transcript of the original genomic DNA


## Scores for c-DNA alignment

- The following scoring scheme solves the problem:
- Constant gap weight
- Heavy penalty for mismatches
- The optimal alignment can be induced to cut up c-DNA to match the exons on the DNA sequence they have originated from


## Diagonal method for finding similar sequences

- If we only want to find the sequences in the database, which are highly similar to a new sequence, we can enforce an additional constraint - the number of insertions, deletions and substitutions not larger than some threshold value $k$
- In this case, we can compute the values of the dynamic programming only in a $2 \mathrm{k}+1$ strip around the main diagonal
- The Miller-Myers algorithm could work well (the edit distance is small)


## When to do the global alignment

o Since the mutational events include multiple genome rearrangements (insertions, deletions, inversions), in addition to mutations, a global alignment of distantly related genomes is of a little biological value
o However, if we would be able to align 2 genomes of the same species (almost identical), we could reveal the sites of polymorphism

## Aligning complete genomes. Challenges

- In order to align 2 genomes, 1,000,000,000 nucleotides each, by a traditional dynamic programming, it will require 10 days of computation on a modern computer system
- The dynamic programming is not easily parallelizable, since we cannot start to compute next value until 3 required previous values have been computed


## The MUMMer algorithm by Delcher et al., 1999

o 3 main algorithmic tools

- Suffix tree
- Longest increasing subsequence
- Dynamic programming
o 3 steps
- Find maximal unique matches - MUMs
- Find the longest sequence of MUMs
- Fill the gaps between MUMs using dynamic programming alignment


## The MUMmer Algorithm. Step 1

o Find maximal unique matches - MUMs

- A MUM is a substring which occurs exactly once in S1 and once in S2, and is not contained in any other such substring
- We can find maximal repeating substrings from a suffix tree
- Among these, take only the nodes which have exactly 2 children - 1 representing some suffix of S1, and 1 representing some suffix of S2
- Output MUMs in form of pairs of start positions (position in S1, position in S2)



## MUMs example

Build a generalized suffix tree for S1 and S2



MUMs example

Maximal
unique
matches:



Maximal
unique
matches:


## ROOT

Output of step 1:
$(6,7)$
$(3,1)$
$(1,2)$

## The MUMmer Algorithm. Step 2

- Find a longest sequence of MUMs, such that positions in both S1 and S2 are increasing and the MUMs are not overlapping
- If we sort MUMs by positions of S1, in order to find a longest sequence of ordered positions in MUMmers of both input strings, we can solve the Longest Increasing Subsequence problem for positions of S2
- The LIS problem can be solved in time $\mathrm{K} \log \mathrm{K}$, where K is a number of MUMs
- Even by a routine Dynamic programming, for finding LIS for a sequence of K positions we need $\mathrm{O}\left(\mathrm{K}^{2}\right)$ operations, and K is much smaller than $N$ - the length of the compared genomes


## Example of DP computation for the Longest Increasing Subsequence

- Suppose the sequence of MUMs is represented by the following pairs:

$$
(1,7)(3,3)(4,8)(5,2)(7,6)(8,9)
$$

Then finding LIS of positions in S 2 is the same as finding a longest common subsequence between $(2,3,6,7,8,9)$ and ( $7,3,8,2,6,9$ )

|  |  | 7 | 3 | 8 | 2 | 6 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 3 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 6 | 0 | 0 | 1 | 1 | 1 | 2 | 2 |
| 7 | 0 | 1 | 1 | 1 | 1 | 2 | 2 |
| 8 | 0 | 1 | 1 | 2 | 2 | 2 | 2 |
| 9 | 0 | 1 | 1 | 2 | 2 | 2 | 3 |

The longest increasing subsequence (not unique) is $(3,8,9)$

And the resulting longest set of MUMs is
$(3,3)(4,8)(8,9)$

## The MUMmer Algorithm. Step 3

- Align the MUMs as exact matches, and fill in the remaining positions by the locally applied dynamic programming
For the above example with S1=gaagcacc and S2=agaaatac the resulting MUMs are $(1,2)(6,7)$

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $s 1$ | g | a | a | g | c | a | c | c |
| s 2 | a | g | a | a | a | t | a | c |


| S1 | - | g | a | a | g | c | a | c | c |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| S2 | a | g | a | a | a | t | a | c | - |

## Heuristic Algorithms

o A heuristic method is an algorithm that gives only approximate solution to a given problem.
o Sometimes we are not able to formally prove that this solution actually solves the problem, but heuristic methods are commonly used because they are much faster than exact algorithms.

## BLAST. Definitions.

## Altschul, Gish, Miller, Myers and Lipman, 1990

- Given two strings S1 and S2, a segment pair is a pair of equal length substrings of S1 and S2, aligned without gaps.
- A locally maximal segment is a segment whose alignment score (without gaps) cannot be improved by extending it or shortening it.
- A maximum segment pair (MSP) in S1 and S2 is a segment pair with the maximum score over all segment pairs in S1, S2.


## BLAST. Outline

o When comparing all the sequences in the database against the query, BLAST attempts to find all the database sequences that when paired with the query contain an MSP above some cutoff score S. We call such a pair, a hi-scoring pair (HSP).

- We choose S such that it is unlikely to find a random sequence in the database that achieves a score higher than $S$ when compared with the query sequence.


## BLAST. Step 1

- Given a length parameter $w$ and a threshold parameter $t$, BLAST finds all the w-length substrings (called words) of database sequences that align with words from the query with an alignment score higher than $t$. Each such hot spot is called a hit in BLAST.
- Instead of requiring words to match exactly, BLAST declares that a word hit has been made if the word taken from the database has a score of at least $t$ when a substitution matrix is used to compare the word from the query. This strategy allows the word size (w) to be kept high (for speed), without sacrificing sensitivity.
- It is usually recommended to set the parameter w to values of 3 to 5 for amino acids, and $\sim 12$ for nucleotides. Thus, $t$ becomes the critical parameter determining speed and sensitivity, and w is rarely varied.
- If the value of $t$ is increased; the number of background word hits will go down and the program will run faster. Reducing $t$ allows more distant relationships to be found.


## BLAST. Step 2

- In the next step, each hit is extended to a locally maximal segment and if its score is above S, i.e. if this sequences pair is HSP, we report these segment
- Since pair score matrices typically include negative values, extension of the initial w-mer hit may increase or decrease the score.
- Accordingly, the extension of a hit can be terminated when the reduction in score (relative to the maximum value encountered) exceeds certain score drop-off threshold.


## Alignments of proteins

o The mutations happen at the level of DNA
o The selection works at the level of proteins

- Aminoacid leucine can be coded by 6 different codons: UUA, UUG, CUU, CUC, CUG, CUA

So if more than 50 percent of nucleotides mutate (UUA->CUG) then this codon still encodes leucine

## Aminoacid substitution matrix

- The unit matrix
- The genetic code matrix:
- the entry equals the number of minimal base substitutions needed to convert a codon of amino acid $i$ to a codon of amino acid $j$. We disregard here the importance of chemical properties of the amino acids, that evidently influence the chances for their successful substitution, like their hydrophobicity, charge or size.

2nd base in codon


For example, distance(Phe, Leu) $=1$ distance(Phe, Gly)=3 distance(Phe,Phe)=0

## Scoring matrix for proteins I. PAM- Point Accepted Mutations

|  | A | R | N | D | C |
| :--- | :--- | :--- | :--- | :--- | :--- |
| A | 9867 | 2 | 9 | 10 | 3 |
| R | 1 | 9913 | 1 | 0 | 1 |
| N | 4 | 1 | 9822 | 36 | 0 |
| D | 6 | 0 | 42 | 9859 | 0 |
| C | 1 | 1 | 0 | 0 | 9973 |

Based on the number of the substitutions from the pairwise alignment of the closely related proteins, which are not more than $1 \%$ different

This is called PAM-1 substitution matrix

## Scoring matrix for proteins II. BLOSUM (BLOck Substitution Matrix)

```
AABCDA...BBCDA
DABCDA.A.BBCBB
BBBCDABA.BCCAA
AAACDA己.DCBCDB
CCBADA3.DBBDCC
AAACAA. . . BBCCC
```

The frequency of the substitutions in the conserved blocks of distantly related proteins, put into a multiple alignment

BLOSUM-65 is the matrix built on the set of proteins which are no more than 65\% similar

## The scores are log scores for

 convenience- Frequency:
- $f(i, j)=\operatorname{count}(i, j) /[c o u n t(i) * \operatorname{count}(j)]$
- The entry of the matrix:
- Score $(i, j)=\log f(i, j)$

